The *Ultimate* Formula Sheet for ACT Math

Fractions, Decimals, & Percentages: (for this section, r is the percent in decimal form)

$Fraction = \frac{part}{whole}$; $Percent = \frac{part}{100}$

Percent Increase or Decrease: $\frac{|old - new|}{old} \times 100\%$

Increase by a percent: multiply by $(1+r)$

Decrease by a percent: multiply by $(1-r)$

Simple Interest: $A = P(1+rt)$

Interest Compounded Annually: $A = P(1+r)^t$

Interest Compounded n times per year: $A = P\left(1+\frac{r}{n}\right)^{nt}$

Rates, Ratios, & Proportions:

General form of a conversion factor: $\left(\frac{ending\_units}{starting\_units}\right)$

Example: $10 feet\left(\frac{12 inches}{1 foot}\right) = 120 inches$

(Concentration of A x Volume of A)
+ (Concentration of B x Volume of B)
= Final concentration (Vol. of A + Vol. of B)

Distance = Rate x Time

Exponents, Roots, & Polynomials:

Multiplication Rule for Exponents: $a^b \cdot a^c = a^{b+c}$

Division Rule for Exponents: $\frac{a^b}{a^c} = a^{b-c}$

Power Rule for Exponents: $(a^b)^c = a^{bc}$

Negative Exponents: $a^{-b} = \frac{1}{a^b}$

Fractional Exponents: $a^{\frac{b}{c}} = \sqrt[c]{a^b}$ or $\left(\sqrt[c]{a}\right)^b$

$i = \sqrt{-1}$; $i^2 = -1$; $i^3 = -i$; $i^4 = 1$

$i^{4n+1} = i$; $i^{4n+2} = -1$; $i^{4n+3} = -i$; $i^{4n} = 1$

Complex Conjugates: $(a+bi)(a-bi) = a^2 + b^2$

Parabolas:

Standard Form: $f(x) = ax^2 + bx + c$

vertex= $\left(-\frac{b}{2a}, f\left(-\frac{b}{2a}\right)\right)$

y-intercept = c;

x-intercepts = $\frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$

Sum of solutions = $\frac{-b}{a}$

Product of solutions = $\frac{c}{a}$

Discriminant = $b^2 - 4ac$; Pos=2 real roots

Zero= 1 real root; Neg=2 imaginary roots

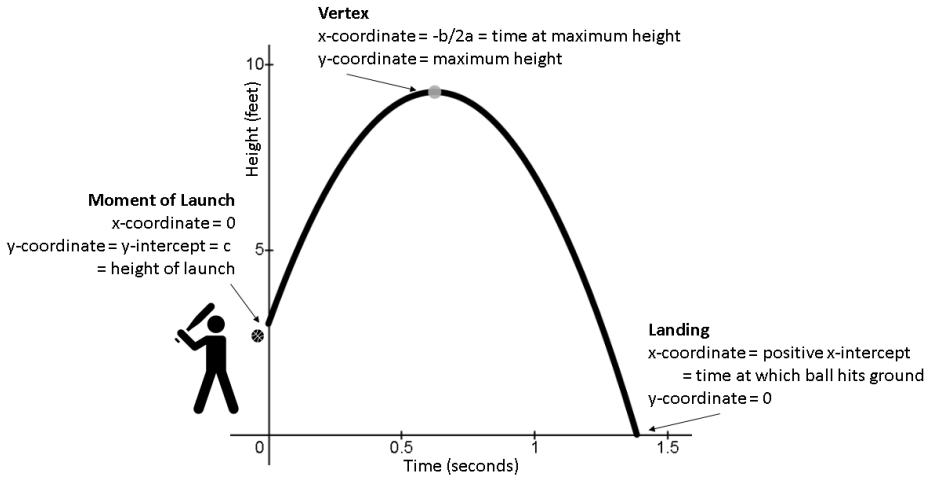
Factored Form: $f(x) = a(x-m)(x-n)$

x-intercepts are m and n

x-coordinate of vertex = $\frac{m+n}{2}$

Vertex Form: $f(x) = a(x-h)^2 + k$

vertex = (h,k)



Difference of Squares: $a^2-b^2=(a+b)(a-b)$

Sum of Cubes: $a^3+b^3=(a+b)(a^2-ab+b^2)$ Difference of Cubes: $a^3-b^3=(a-b)(a^2+ab+b^2)$

Perfect Square Trinomial: $a^2+2ab+b^2=(a+b)^2$ and $a^2-2ab+b^2=(a-b)^2$

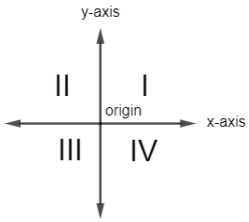
Completing the Square: $x^2+bx+\left(\frac{b}{2}\right)^2=\left(x+\frac{b}{2}\right)^2$

Graphing Lines:

Slope Formula: $m=\frac{y_2-y_1}{x_2-x_1}$

Slope of horizontal line = 0

Slope of vertical line = undefined



Standard Form: $Ax+By=C$

Slope-Intercept Form: $y=mx+b$

Point-Slope Form: $y-y_1=m(x-x_1)$

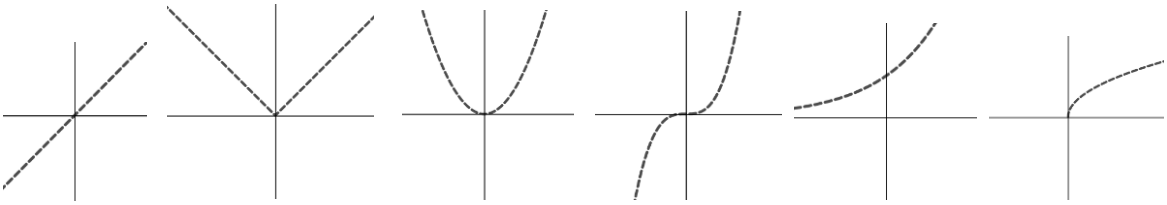
Distance Formula: $d=\sqrt{(x_2-x_1)^2+(y_2-y_1)^2}$

Midpoint Formula: $M=\left(\frac{x_1+x_2}{2},\frac{y_1+y_2}{2}\right)$

Parallel lines: equal slopes

$\perp$ Lines: slopes are opposite reciprocals

Parent Graphs & Transformations:



$y=x$ $y=|x|$ $y=x^2$ $y=x^3$ $y=a^x$ $y=\sqrt{x}$

Transformation	Visual effect
$f(x)+k$	Shift up by k units
$f(x)-k$	Shift down by k units
$f(x+h)$	Shift left by h units
$f(x-h)$	Shift right by h units
$-f(x)$	Reflect over the x axis (flip upside down)
$cf(x)$	Stretch vertically by a factor of c (becomes skinnier)
$\frac{1}{c}f(x)$	Shrink vertically by a factor of c (becomes fatter)

Data & Probability:

$$Average = \frac{sum\_of\_items}{number\_of\_items}$$

Median = center data point

Mode = most frequent data point

Range = maximum – minimum

$$Probability = \frac{desired\_outcomes}{possible\_outcomes}$$

Probability that independent events A and B will both happen: $P(A \cap B) = P(A) \times P(B)$

Probability that either A or B will happen: $P(A \cup B) = P(A) + P(B) - P(A \cap B)$

Expected Value: $E(x) = \sum_{i=1}^{n} x_i \bullet P(x_i)$

Angles:

Vertical ∠' s are ≅

∠'s that form a linear pair are supplementary (add up to 180°)

∠' s that form a circle add up to 360°

When ∥ lines are cut by a transversal, all acute ∠' s are ≅ and all obtuse ∠' s are ≅

Triangles:

Area of a Triangle: $A = \frac{1}{2}bh$

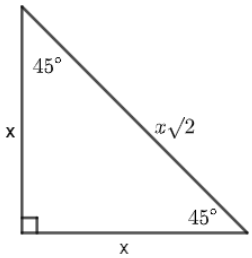
The three ∠' s of a Δ add up to 180°

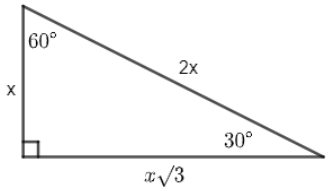
An exterior ∠ is equal to the sum of the two remote interior ∠' s

Pythagorean Theorem: $a^2 + b^2 = c^2$

Pythagorean Triples: 3-4-5 and 5-12-13

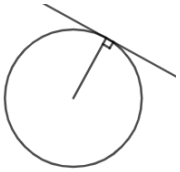
Special Right Triangles:





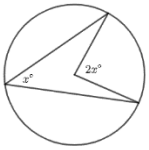
Circles:

Area of a Circle: $A = \pi r^2$



A radius and tangent make a right ∠

Circumference of a Circle: $C = 2\pi r$



A central ∠ is double the inscribed ∠

$\frac{x}{360} = \frac{arc}{circumference}$ and $\frac{x}{360} = \frac{sector}{area\_of\_circle}$

where x = central angle

Polygons: (for this section, n is the number of sides)

Area of a Rectangle: $A = lw$

Area of a trapezoid: $A = \frac{1}{2}(b_1 + b_2)h$

Sum of the exterior angles: 360°

Sum of the interior angles: $180(n-2)$

One int. ∠ of a regular polygon: $\frac{180(n-2)}{n}$

# of diagonals: $\frac{n(n-3)}{2}$ (convex only)

Properties of Parallelograms:

1. Opp sides are ∥ and ≅
2. Opp ∠' s are ≅
3. Consec ∠'s are supplementary
4. Each diagonal forms a pair of ≅Δ's
5. Diagonals bisect each other

➔If they are ≅ it is a rectangle

➔If they are ⊥ it is a rhombus

6. $Area = base \times height$

Solids:

Volume of a Rectangular Prism (Box): $V = lwh$

Surface Area of a Box: $SA = 2(lw + lh + wh)$

Volume of a Cylinder: $V = \pi r^2 h$

Surface Area of a Cylinder: $SA = 2\pi r^2 + 2\pi rh$

Volume of a Sphere: $V = \frac{4}{3}\pi r^3$

Volume of a Cone: $V = \frac{1}{3}\pi r^2 h$

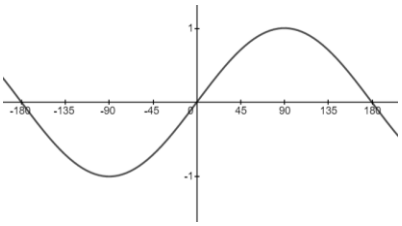
Volume of a Pyramid: $V = \frac{1}{3}lwh$

Trigonometry:

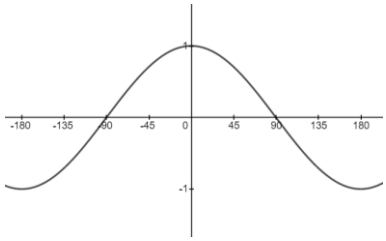
$\sin = \frac{opp}{hyp}$ $\cos = \frac{adj}{hyp}$ $\tan = \frac{opp}{adj}$ $\csc(x) = \frac{1}{\sin(x)}$ $\sec(x) = \frac{1}{\cos(x)}$ $\cot(x) = \frac{1}{\tan(x)}$

360°=2π radians $\tan x = \frac{\sin x}{\cos x}$ $\sin^2 x + \cos^2 x = 1$ $\sin(x) = \cos(90 - x)$

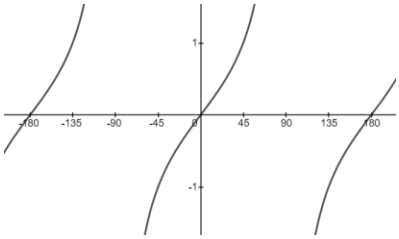
Law of Sines: $\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$ Law of Cosines: $a^2 = b^2 + c^2 - 2bc \cdot \cos(A)$



$y = \sin(x)$



$y = \cos(x)$



$y = \tan(x)$

If $y = A\sin(Bx - C) + D$ (also for cos, csc, and sec)

Amplitude: $|A|$ Period: $\frac{2\pi}{B}$ Phase Shift: $\frac{C}{B}$ Vertical Shift: D

If $y = A\tan(Bx - C) + D$ (also for cot)

Amplitude: none Period: $\frac{\pi}{B}$ Phase Shift: $\frac{C}{B}$ Vertical Shift: D

Sequences and Series: where a_1 = first term, n = number of terms, d = common difference, r = common ratio

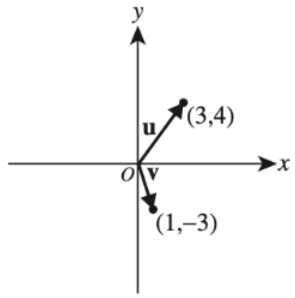
Arithmetic sequence: $a_n = a_1 + (n-1)d$ Geometric sequence: $a_n = a_1 r^{n-1}$

Sum of an arithmetic series: $S_n = \frac{n}{2}(a_1 + a_n)$ Sum of a geometric series: $S_n = \frac{a_1(r^n - 1)}{r - 1}$

Logarithms:

If $\log_b a = x$, then $b^x = a$ $\log_b a = \frac{\log a}{\log b}$

Vectors:



$\boldsymbol{u} =< 3, 4 >$

$\boldsymbol{v} =< 1, -3 >$

$\boldsymbol{u} + \boldsymbol{v} =< 4, 1 >$

$\boldsymbol{u} - \boldsymbol{v} =< \mathbf{2}, \mathbf{7} >$

Matrix Addition: Only possible when rows of first = rows of second AND columns of first = columns of second.

$$\begin{bmatrix} \boldsymbol{A} & \boldsymbol{B} \\ \boldsymbol{C} & \boldsymbol{D} \end{bmatrix} + \begin{bmatrix} \boldsymbol{E} & \boldsymbol{F} \\ \boldsymbol{G} & \boldsymbol{H} \end{bmatrix} = \begin{bmatrix} \boldsymbol{A+E} & \boldsymbol{B+F} \\ \boldsymbol{C+G} & \boldsymbol{D+H} \end{bmatrix}$$

Matrix Multiplication: Only possible when columns of first = rows of second

$$\begin{bmatrix} \boldsymbol{A} & \boldsymbol{B} \\ \boldsymbol{C} & \boldsymbol{D} \end{bmatrix} \times \begin{bmatrix} \boldsymbol{E} & \boldsymbol{F} \\ \boldsymbol{G} & \boldsymbol{H} \end{bmatrix} = \begin{bmatrix} \boldsymbol{AE+BG} & \boldsymbol{AF+BH} \\ \boldsymbol{CE+DG} & \boldsymbol{CF+DH} \end{bmatrix}$$

Conic Sections:

Circle: $(x-h)^2+(y-k)^2=r^2$, where (h,k) is the center and r is the radius

Ellipse: $\frac{(x-h)^2}{a^2}+\frac{(y-k)^2}{b^2}=1$ where (h,k) is the center, 2a is the horizontal axis, and 2b is the vertical axis

Horizontal Ellipse: $a^2=b^2+c^2$ Vertical Ellipse: $b^2=a^2+c^2$ where c is the distance from center to focus

Horizontal Hyperbola: $\frac{(x-h)^2}{a^2}-\frac{(y-k)^2}{b^2}=1$ Vertical Hyperbola: $\frac{(y-k)^2}{a^2}-\frac{(x-h)^2}{b^2}=1$